The Nuclear Structure of $^{182}\text{W}$ Isotope

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Abstract:

In this work, the energy levels and reduced electric transition for $^{182}\text{W}$ isotope had been studied using interacting boson model-1. Also, the nucleus shape was determined through studying potential energy surface; the square of rotational energy and the moment of inertia were calculated. In the present research, the spin and parity was found for some energy levels which not determined practically, such as the levels $(1.6234, 1.9592, 1.7563)$ MeV with spin and parity $(5^+\, , 3^+\, \text{and} \, 6^+)$ respectively. According to the (IBM-1) it was found that $^{182}\text{W}$ isotope involved in symmetry of SU(3).

التركيب النووي لنظير $^{182}\text{W}$

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الخلاصة :

أجري العمل دراسة خصائص مستويات الطاقة لنظير $^{182}\text{W}$ واحتمالات الانتقالات الكهربائية المختلطة $^{182}\text{W}$. واستعمال نموذج اليوثرات المتفاعلة -1. كما حدد شكل النواة من خلال دراسة طاقة جهد السطح وحساب $B(E2)$ مربع الطاقة الدورانية وعزم القصور الذاتي. كما جرى في هذا البحث تأكيد البرم والتماثيل لبعض مستويات الطاقة غير المحدودة عملياً، حيث تم تحديد المستويات $(1.6253, 1.9592, 1.6234)$ MeV و $(5^+, 3^+, 6^+)$ على التوالي. ووفقاً لنموذج (IBM-1) $^{182}\text{W}$ ينتمي إلى التماثيل SU(3).
1- Introduction:

Some of the scientist studied the nuclear structure of (TUNGESTEN) isotopes through using different theoretical and practical models so some of the researchers were dealing with this subject. In 1981 J.P.Delaoche et.al [1] studied the nuclear deformation, moment's radius of 182-186W isotopes from fast neutron scattering .S.T.Hsieh and M.M.K.Yen [2] in 1987 studied the high spin stets of 172W nuclide by using the core excited interacting boson model (IBA) ,in addition to energy levels. They calculated transition quadrupole moments . In 1994 L.M.Chen [3] studied the energy levels ,transition energy B(E2) and the effective moments of inertia of 166-176W the (even-even) isotopes by using the interacting boson- fermions model. In 1996 P.Navratil and et.al [4] studied (E2/M1) mixing ratios (g) factor and summed (M1) strength even 182-186W isotopes by using (IBM-2) . In 2001 D.L.Balabanski and et.al [5] studied quadrupole moment of 179W isotope by using (level mixing spectroscopy method). And in (2008) K.Alataya [6] studied the energy levels , reduced electric transition , potential energy surface; the square of rotational energy and the moment of inertia for 184W isotope by using (IBM-1) .

The IBM-1 Model:
The IBM-1 model was used to the described low lying collective state of energy levels in (even-even) nucleus which can be described by (s) bosons when (J= 0\(^+\)) and (d) bosons when (J= 2\(^+\))

The SU (3) dynamic symmetry based on the boson energy (\(\varepsilon\)) is smaller than the reaction potential (V) , (V >> \(\varepsilon\)) . Both of the reaction electric quadrupole moment (Q,Q) and the reaction angular momentum (L,L) are controlled on the Rotational limit SU(3) .therefore the general Hamilton formula for this limit is[7] :

\[
H^1 = a_1 L^2 + a_2 Q^2
\]

and the equation of eigen value to Hamilton is given by [8] :

\[
E[N,(\lambda,\mu),K,L,M] = \frac{a_2}{2} (\lambda^2 + \mu^2 + \lambda \mu + 3(\lambda + \mu) + (a_1 - \frac{3a_2}{8}).L(L-1)........(2)
\]

where:

\{ (\lambda,\mu),K,L,M \} the quantum numbers, but (\lambda,\mu) determined the Rotational limit SU(3) state.

The transition operator \(T_m^{(E_i)}\) for these limits were given by the following formula [7]:

\[
T_m^{(E_i)} = \alpha_2^2 \left[ (d^\dagger s + s^\dagger d)^{(2)}_m - \frac{\sqrt{L}}{2} (d^\dagger d)^{(2)}_m \right] ............(3)
\]

The selection rules [9,10] are ( \(\Delta \lambda = 0, \Delta \mu = 0\)) , the Equation which used to evaluate the reduced transition probability B(E2) is [11] :

\[
B(E_2, L+2 \rightarrow L) = \alpha_2^3 \frac{3}{4} \frac{(L+2)(L+1)}{(2L+3)(2L+5)} (2N-L)(2N+L+3).............(4)
\]

\[
B(E_2, 2_1^+ \rightarrow 0^+ ) = \alpha_2 \frac{N}{5} (2N+3).........................(5)
\]

The formula which was used for determining the electric quadrupole moment (Q) to these limits equal :[10]
The relation between \((\alpha^2)\) and \((\beta^2)\) for these limits is:

\[
\beta_2 = -\frac{\sqrt{7}}{2} \alpha_2 \quad \text{.........}(8)
\]

Where ((\(\alpha^2\)) and (\(\beta^2\))) parameters used for determining the reduced transition probability.

2- Potential energy surface:

The potential energy surface function of total number of bosons and deformation factors of \((\beta, \gamma)\) were calculated from the equation (9).[8,9]

\[
V(N, \beta, \gamma) = \frac{\langle N, \beta, \gamma | \hat{H} | N, \beta, \gamma \rangle}{\langle N, \beta, \gamma | N, \beta, \gamma \rangle} \quad \text{.........}(9)
\]

By derive the equation (9) with respect to \((\alpha, \beta)\) we get :[8,9]

\[
V(N, \beta, \gamma) = \frac{N}{1 + \beta^2} (e_s + e_d \beta^2) + \frac{N(N-1)}{(1 + \beta^2)} \left( A_1 \beta^2 + A_2 \beta^3 \cos 3 \gamma + A_3 \beta^2 + A_4 \right) \quad \text{.........}(10)
\]

\(N\) : total number of bosons.
\(\beta\) : Magnitude of Nuclear Deformation , takes the values (0 to 2.4).
\(\gamma\) : Asymmetry Angle , takes the values (0° to 60°).
\(A_1, A_2, A_3, A_4\) : Parameters of potential surface .

The Deformed Nuclei dependeds on \((\beta, \gamma)\) factors .when \(\beta=0\) the Nuclei is spherical and when \(\beta>0\) the Nuclei is Deformed , otherwise when \(\gamma =0°\) the Deformations nuclei are spherical of Prelate Shape and when \(\gamma =60°\) the Deformations nuclei are spherical of Oblate Shape.

3-Rotational Motion Nucleus and Moment of Inertia:

The formulae for calculating all the square of rotational energy and the moment of inertia are: [12]

\[
\hbar^2 \omega^2 = (L^2 - L + 1) \left( \frac{E(L \to L-2)}{2L-1} \right)^2 \quad \text{.........}(11)
\]

\[
\frac{2\nu}{\hbar^2} = \frac{4L-2}{E(L \to L-2)} \quad \text{.........}(12)
\]

4-Calculations:

When studing the nuclear properties of \(^{182}\)W isotope it was found that this isotope determine the rotational limit, depending on the results which were calculated by using program interacting boson model -1 (IBM-1) and compared with the Experimental result, in addition to the shape of potential energy surface.

The nuclear properties which we studied in this research are energy levels, the reduced transition probability, the potential energy surface, the square of rotational energy and the moment of inertia.
5- Calculation of energy levels:
The experimental values of the energy levels show the natural rotation, means the $^{182}$W isotope determined to rotational limit SU(3). The parameter was given coincidences with experimental value [13,14,15,16] show in table (1).

Table (1) parameters used in this program for calculation of energy levels $^{182}$W in MeV

<table>
<thead>
<tr>
<th>isotope W$^{182}$</th>
<th>E.p.s</th>
<th>P.P</th>
<th>L.L</th>
<th>Q.Q</th>
<th>T3.T3</th>
<th>T4.T4</th>
<th>CH$_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0106</td>
<td>-0.0163</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1.100</td>
<td></td>
</tr>
</tbody>
</table>

Either the theoretical of the energy levels were calculated by using (IBM-1) model and comparison with the Experimental of the energy levels shown in table (2).

Table (2) the energies of levels in $^{182}$W found experimentally as compared with IBM-1 for positive parity calculation:

<table>
<thead>
<tr>
<th>J$^+$</th>
<th>Theo.(MeV)</th>
<th>Exp.(MeV)[12]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0$^+_1$</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>2$^+_1$</td>
<td>0.09953</td>
<td>0.1001</td>
</tr>
<tr>
<td>3$^+_1$</td>
<td>1.24062</td>
<td>1.331</td>
</tr>
<tr>
<td>4$^+_1$</td>
<td>0.33174</td>
<td>0.3294</td>
</tr>
<tr>
<td>5$^+_1$</td>
<td>1.54143</td>
<td>(1.6234)</td>
</tr>
<tr>
<td>6$^+_1$</td>
<td>0.6959</td>
<td>0.6805</td>
</tr>
<tr>
<td>8$^+_1$</td>
<td>1.19399</td>
<td>1.1445</td>
</tr>
<tr>
<td>10$^+_1$</td>
<td>1.82384</td>
<td>1.7121</td>
</tr>
<tr>
<td>0$^+_2$</td>
<td>1.13543</td>
<td>1.1357</td>
</tr>
<tr>
<td>2$^+_2$</td>
<td>1.14032</td>
<td>1.2214</td>
</tr>
<tr>
<td>2$^+_3$</td>
<td>1.23526</td>
<td>1.2574</td>
</tr>
<tr>
<td>3$^+_2$</td>
<td>2.20150</td>
<td>(1.9592)</td>
</tr>
<tr>
<td>4$^+_2$</td>
<td>1.37424</td>
<td>1.4428</td>
</tr>
<tr>
<td>6$^+_2$</td>
<td>1.74151</td>
<td>(1.7563)</td>
</tr>
</tbody>
</table>
Fig (1) the energy levels in $^{182}$W found experimentally as compared with IBM-1 for positive parity calculation

7- Calculation the reduced transition probability $B(E_2)$:

The values of reduced transition probability $B(E_2)$ were calculated by using program (IBMT-1), depend on the value of parameters ($\beta_2$, $\alpha_2$) which was it calculated from equations(5,8). In this study determination these parameters depends on the experimental value for transition $B(E_2, 2_1^+ \rightarrow 0_2^+)$. The parameters which were used in the (IBMT1) program ($E_{2SD}$ & $E_{2DD}$) can be calculated as follow and shown in table(3):

\[ E_{2SD} = \alpha_2 \] \[(9)\]  
\[ E_{2DD} = \sqrt{5}\beta_2 \] \[(10)\]

In addition to, the value of ($\alpha_2$) can be found after calculated transition $B(E_2)$ from the following equation \[(17)\]

\[ B(E_2) = \frac{56.57}{E_{2SD}^5 t_{1/2} \left(1 + \alpha_{tot}^2\right)}(e^2b^2) \] \[(11)\]

Where:
- $E_\gamma$: gamma ray transition energy in (KeV)
- $t_{1/2}$: half life for the level ($2_1^+$).
- $\alpha_{tot}$: Total internal conversion coefficient which can be found from the table [18].

The table (4) shows the compare of experimental and theoretical values of reduced transition probability $B(E_2)$ of $^{182}$W.

**Table (3)** show the parameters ($E_{2SD}$ & $E_{2DD}$) used in (IBMT-1) program for calculating reduced transition probability $B(E2)$ of $^{182}$W.

<table>
<thead>
<tr>
<th>isotope</th>
<th>$E_{2SD}$</th>
<th>$E_{2DD}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{182}$W</td>
<td>0.10486</td>
<td>-0.31019</td>
</tr>
</tbody>
</table>
Table (4) the values of theoretical $B(E2)$ in $^{182}\text{W}$ by using (IBMT-1)Code as compared with the values of experiential $B(E2)$

<table>
<thead>
<tr>
<th>$I_i^n \rightarrow I_f^n$</th>
<th>$B(E2)\downarrow e^2b^2$ Theo.</th>
<th>$B(E2)\downarrow e^2b^2$ exp[12]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2_1^+\rightarrow 0_1^+$</td>
<td>0.8404709</td>
<td>0.84</td>
</tr>
<tr>
<td>$2_1^+\rightarrow 0_2^+$</td>
<td>0.0437639</td>
<td></td>
</tr>
<tr>
<td>$2_2^+\rightarrow 0_1^+$</td>
<td>0.0949430</td>
<td></td>
</tr>
<tr>
<td>$2_2^+\rightarrow 0_2^+$</td>
<td>0.0036000</td>
<td></td>
</tr>
<tr>
<td>$2_3^+\rightarrow 0_2^+$</td>
<td>0.07143912</td>
<td></td>
</tr>
<tr>
<td>$2_3^+\rightarrow 0_3^+$</td>
<td>0.0149709</td>
<td></td>
</tr>
<tr>
<td>$2_3^+\rightarrow 0_4^+$</td>
<td>0.0022427</td>
<td></td>
</tr>
<tr>
<td>$2_4^+\rightarrow 0_3^+$</td>
<td>0.4175120</td>
<td></td>
</tr>
<tr>
<td>$2_4^+\rightarrow 0_4^+$</td>
<td>0.0406843</td>
<td></td>
</tr>
<tr>
<td>$4_1^+\rightarrow 2_1^+$</td>
<td>1.1183700</td>
<td></td>
</tr>
<tr>
<td>$4_1^+\rightarrow 2_2^+$</td>
<td>0.0017174</td>
<td></td>
</tr>
<tr>
<td>$4_1^+\rightarrow 2_3^+$</td>
<td>0.0717113</td>
<td></td>
</tr>
<tr>
<td>$4_2^+\rightarrow 2_1^+$</td>
<td>0.0473603</td>
<td></td>
</tr>
<tr>
<td>$4_2^+\rightarrow 2_2^+$</td>
<td>0.3125263</td>
<td></td>
</tr>
<tr>
<td>$4_2^+\rightarrow 2_3^+$</td>
<td>0.9316370</td>
<td></td>
</tr>
</tbody>
</table>

8-Calculation of potential energy surface:

The potential energy surface was calculated after determining the parameters of Hamilton function operator that specialized for $^{182}\text{W}$. Table (5) shows the parameters which are used in (IBMP-1) program for calculating potential energy surface $V(\beta,\gamma)$.

Table (5) parameters used in this program for calculating potential energy surface for $^{182}\text{W}$ in MeV

<table>
<thead>
<tr>
<th>isotope</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
<th>ES</th>
<th>ED</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{182}\text{W}$</td>
<td>-0.006</td>
<td>-0.038</td>
<td>-0.065</td>
<td>0.000</td>
<td>0.081</td>
<td>0.028</td>
</tr>
</tbody>
</table>

The potential energy surface gives the last shape of nucleus. It is agreement with Hamilton function [21] of two parameters ($\beta,\gamma$). Fig (1) shows $^{182}\text{W}$ isotope that rotational limit Su(3) compared with the paper [8].
9-Calculation of the square of rotational energy and the moment of inertia.

The square of rotational energy and the moment of inertia could be calculated from equations (10, 11) after finding the energy levels using (IBM-1) program and angular moment to all energy levels. Fig (2) and (3) show comparison between the theoretical values and the experimental values [12] for all square rotational energy and the moment of inertia.

Fig (3) shows the comparison between the theoretical values and the experimental values for the square of rotational energy.
Fig (4) shows the comparison between the theoretical values and the experimental values for the moment of inertia.

10 Result and Dissection:

Table (1) shows parameters using in this program for the calculation of energy levels .it was found the (Q.Q) and (L.L) dominate on other parameters .also there are three indices refers to $^{182}$W SU (3) The energy levels , Reduced electric transition probability and potential energy surface as follow :

1) Energy levels: the ratio of energy levels $E_{0^+_2}^+, E_{4^+_1}^+, E_{8^+_1}^+, E_{6^+_1}^+$

Refers to a good approach to SU (3) after comparison with experimentally value [9,20].

2) Reduced electric transition probability: the levels decay $2^+_1$ to $0^+_1$ and $4^+_1$ to $2^+_1$ in one band and don’t decay between bands [21].

3) Potential energy surface: The contour shape in fig (1) shows this isotopes SU (3) as comported with experimentally figure of SU(3) [11].

References:
11- K. S. Krane , "Introductory Nuclear Physics", Ed. Halliday